DETERMINING THE QUANTITY OF MATERIAL WORN AWAY FROM A WALL IN A TURBULENT BOUNDARY LAYER AS A CONSEQUENCE OF CHEMICAL EROSION WHEN AN INERT INHOMOGENEOUS GAS IS ADDED BY INJECTION THROUGH THE ENTRAINED SURFACE UNDER CONDITIONS OF SUBSTANTIAL NONISOTHERMICITY

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We propose an analytical method of calculating the entrainment of wall material when an inert gas is additionally injected to that wall in a turbulent boundary layer. The calculation is carried out for the case of carbon entrainment when additional argon and helium are injected through a carbon wall when b_g = const under conditions of substantial nonisothermicity.

In equipment-building practice we frequently encounter cases in which a material streamlined by a flow is decomposed as a consequence of the thermal effect of a high enthalpy. The gaseous components in this case are injected into the boundary layer, and the solid residue is removed by means of chemical erosion.

It is the purpose of this paper to determine the magnitude of wear which is experienced by the surface through which a different inert gas is injected.

1. For a chemically reacting gas the equations of energy and continuity for the i-th component of the effective binary mixture, if these are written in terms of the total enthalpy and the reduced concentration, will be of the conventional form [1, 2]. With consideration of the continuity equation for a flat boundary layer we reduce the equation of mass – after simple transformations, to the integral relationship

$$\frac{d(\mathrm{Re}_g^{**}\Delta K_i^0)}{\Delta K_i^0\, d\bar{x}} - \mathrm{Re}_L \frac{j_w}{\rho_0 u_0} = \mathrm{Re}_L \frac{\rho D_{12}\left(\frac{\partial K_i^0}{\partial y}\right)_w}{\rho_0 u_0 \Delta K_i^0}. \tag{1}$$

Here Re_g^{**} and Re_L are characteristic Reynolds numbers written, respectively, for the mass-loss thickness δ_g^{**} and the characteristic linear dimension L:

$$\delta_g^{**} = \int\limits_0^\delta \frac{\rho u}{\rho_0 u_0} \frac{(K_i^0)_0 - K_i^0}{(K_i^0)_0 - (K_i^0)_w}\, dy;$$

$$\Delta K_i^0 = (K_i^0)_0 - (K_i^0)_w; \quad \bar{x} = x/L.$$

The mass flow of the wall is written in the form of the sum of the transverse flow generated by removal of wall material and the mass flow of the gas additionally injected through the wall, i.e.,

$$j_w = j_{\mathrm{wall}} + j_G \tag{2}$$

The overall wall-permeability parameter, in conjunction with Eq. (2), is expressed by the equation

$$b_1 = b_{\mathrm{wall}} + b_G \tag{3}$$

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In analogy with heat transfer, we will denote the right-hand member of the integral relationship (1) by the Stanton diffusion number:

$$\frac{\rho D_{12}\left(\frac{\partial K_i^0}{\partial y}\right)_w}{\rho_0 u_0 \Delta K_i^0} = \mathrm{St}_g = \frac{j_w}{\rho_0 u_0 b_1} = \frac{j_{\mathrm{wall}}}{\rho_0 u_0 b_c} = \frac{j_G}{\rho_0 u_0 b_G}. \tag{4}$$

From the mass-balance equation for the i-th component at the wall, with consideration of the identities (4), we can determine the wall-permeability parameters:

$$b_{\mathrm{wall}} = (K_{\mathrm{wall}}^0)_w (1 + b_1), \tag{5}$$

$$b_G = (K_G^0)_w (1 + b_1), \tag{6}$$

$$b_1 = \frac{(K_O^0)_0 - (K_O^0)_w}{(K_O^0)_w}. \tag{7}$$

Here K_{wall}^0 is the reduced concentration of the wall material; $(K_G^0)_w$ is the concentration of the injected gas at the wall; K_0^0 is the reduced concentration of that component of the gas flow that has not penetrated the wall, and which enters into chemical interaction with the wall material.

For example, in the region of diffusion reaction between the carbon and the oxidizer, the basic product of the reaction is carbon monoxide:

$$\mathrm{C} + \mathrm{O} \rightarrow \mathrm{CO}. \tag{8}$$

From the stoichiometric relationship of (8) we have

$$(K_O^0)_w = \frac{16}{12} (K_c^0)_w. \tag{9}$$

From (7) in conjunction with (9) we obtain the expression for the wall-permeability parameter which is a consequence of chemical erosion in the case of reaction (8):

$$b_{\mathrm{wall}} = \frac{3}{4} (K_O^0)_0. \tag{10}$$

The permeability parameter b_{wall} for the diffusion region of reaction is thus a constant determined by the oxidation potential of the basic flow and the nature of the material being removed; it is independent of the magnitude of the additional inert inhomogeneous gas injected through the surface being worn away.

2. In analogy with heat transfer, we adopt the assumption that the mass-transfer law is not affected by perturbation factors (nonisothermicity, the transverse flow generated by chemical reactions and injection) and for standard conditions it is written in the form [3, 4] (in the region $200 < \mathrm{Re}_g^{**} < 10^4$)

$$\mathrm{St}_{0g} = \frac{0.0128}{\mathrm{Re}_g^{**0.25}\,\mathrm{Sc}^{0.75}} \left(\frac{\mu_w}{\mu_0}\right)^{0.25}. \tag{11}$$

Integral equation (1), with consideration of definition (4), is transformed to

$$\frac{d(\mathrm{Re}_g^{**} \Delta K_i^0)}{\Delta K_i^0\, dx} = \mathrm{Re}_L\, \mathrm{St}_{0g}\, \Psi_h (1 + b_1). \tag{12}$$

Here $\Psi_h = \mathrm{St}_g/\mathrm{St}_{0g}$ is the relative coefficient of heat and mass transfer, by means of which we take into consideration the effect of nonisothermicity, injection, etc.

For the inhomogeneous injection of gases which result in chemical reactions at the surface under conditions of substantial nonisothermicity, the relative mass-transfer law is defined as

$$\Psi_h = \left(\frac{2}{\sqrt{\psi_1}+1}\right)^2 \left(\frac{2}{\sqrt{\psi_2}+1}\right)^2 \left(\frac{\sqrt{\psi_3}+1}{2}\right)^2 \left(1 - \frac{b_1 \Psi_h}{b_{\mathrm{cr}}}\right)^2. \tag{13}$$

Here

$$\psi_1 = h_w/h_0;\quad \psi_2 = M_0/M_w;\quad \psi_3 = c_{pw}/c_{p0}.$$

The critical injection parameter b_{cr} is also a function of ψ_1, ψ_2, and ψ_3.

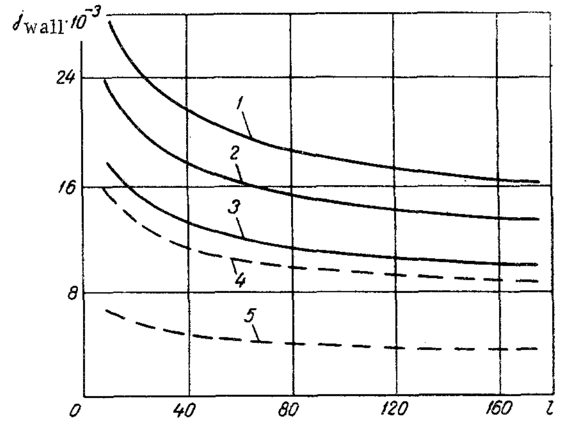

Fig. 1

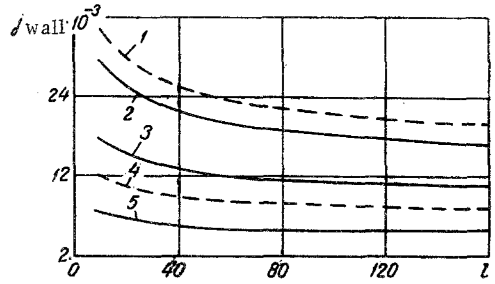

Fig. 2

Fig. 1. Graphite erosion in the presence of argon and helium injection: 1) without additional injection; 2, 3) with additional injection of argon at $b_G = 0.5$ and 2.0, respectively; 4, 5) with additional injection of helium at $b_G = 0.5$ and 2.0, respectively.

Fig. 2. Effect of nonisothermicity and injection on the magnitude of graphite erosion: 1) effect of nonisothermicity on injection of argon, $b_G = 2.0$; 2) graphite erosion without additional gas injection; 3) combined effect of nonisothermicity and injection for argon, $b_G = 2.0$; 4) effect of nonisothermicity on erosion with injection of helium when $b_G = 2.0$; 5) effect of nonisothermicity and injection for helium with $b_G = 2.0$.

To determine the magnitude of the Stanton diffusion number, we have to find Re_g^{**}.

The solution of (12), with consideration of (11), has the following form [3]:

$$\text{Re}_g^{**} = \left[\frac{0.016\,\text{Re}_L}{\text{Sc}^{0.75}\,(\Delta K_i^0)^{1.25}} \int_0^{\bar{x}} \Psi_h (1 + b_1)\,(\Delta K_i^0)^{1.25} \left(\frac{\mu_w}{\mu_0} \right)^{0.25} d\bar{x} \right]^{0.8}. \tag{14}$$

To process the experimental data, it is convenient to use the method of local simulation [3]. From the integral relationship (1), if it is written in terms of the entrained component of the wall material, with consideration given to its mass balance at the wall, we obtain

$$\text{Re}_g^{**} = \frac{\int_0^x j_{\text{wall}}\,dx}{g\mu_0\,(K_{\text{wall}}^0)_w}. \tag{15}$$

3. Let us examine the case of the wearing down of graphite heated to 2000°K. An inert gas (argon, helium, etc.) is injected through the graphite. The injection obeys the law $b_G = \text{const}$ over the length of the plate. The main flow is air at 290°K.

In this case, we see from (3) and (10), with consideration of (5) and (6), that ΔK_i^0 and Ψ_h are constant over the length of the boundary layer.

Expression (14) for Re_g^{**} is thus simplified to

$$\text{Re}_g^{**} = \left[0.016\,\text{Re}_x\,\text{Sc}^{-0.75}\,\Psi_h (1 + b_1) \left(\frac{\mu_w}{\mu_0} \right)^{0.25} \right]^{0.8}. \tag{16}$$

With consideration of (16), we write the dimensionless Stanton number on the basis of (11) in the form

$$\text{St}_{0g} = 0.029\,\text{Re}_x^{-0.2}\,\text{Sc}^{-0.6}\,\Psi_h^{-0.2} (1 + b_1)^{-0.2} \left(\frac{\mu_w}{\mu_0} \right)^{0.2}. \tag{17}$$

The magnitude of the mass entrainment of the wall material is found from the equation

$$j_{\text{wall}} = 0.029\,b_{\text{wall}} \gamma_0 u_0\,\text{Re}_x^{-0.2}\,\text{Sc}^{-0.6}\,\Psi_h^{0.8} (1 + b_1)^{-0.2} \left(\frac{\mu_w}{\mu_0} \right)^{0.2}. \tag{18}$$

For air the oxidation potential is given by $b_{wall} = 0.173$. At a wall temperature about 1700°K we have a diffusion regime for the reaction between the oxygen and the carbon. The primary product of reaction is CO.

From the stoichiometric relationship of reaction (8), in conjunction with identity (5), we find the weight content of the CO product at the wall, i.e.,

$$(K_{\mathrm{CO}})_w = \frac{28}{12}\,\frac{b_{\mathrm{wall}}}{1 + b_1}\,. \tag{19}$$

The concentration of the inert component of the basic flow (nitrogen in this case), diffusing toward the wall, is determined from the relationship

$$(K^0_{\mathrm{in}})_w = \frac{(K^0_{\mathrm{in}})_0}{1 + b_1}\,. \tag{20}$$

The weight fraction of the injected gas is found from (6).

The molecular weight and the heat capacity of the gas mixture at the wall temperature are calculated with the familiar formulas for a multicomponent mixture.

The viscosity of the gas mixture at the wall – at the temperature of the wall – can be determined from the Mann formula [5], i.e.,

$$\nu_w = \left[\sum_{i=1}^{n} \frac{n_i}{\nu_i}\right]^{-1}. \tag{21}$$

Here ν is the kinematic viscosity; n_i are the molar fractions of the components making up the gas mixture at the wall.

The Schmidt number Sc is taken from the parameters of the basic flow, at the temperature of that flow.

Figure 1 shows the results obtained in calculating the erosion of a graphite surface when additional quantities of argon and helium are injected through that surface. We see that on injection of argon with $b_G = 2$, and the identical wall temperature, the erosion of the carbon at mass flow rate of 100 $\mathrm{kg/m^2 \cdot sec}$ for the main flow diminishes, on the average, by a factor of 1.8, while with injection of helium it diminishes by a factor of 4.8 in comparison with the magnitude of the erosion without injection.

On injection of helium, when $b_G = 20$, the graphite erosion is reduced by a factor of almost 30, with the relative average flow rate for the injected gas on a plate 200 mm in length (referred to $\gamma_0 u_0$) amounting to $\sim 0.8\%$ in this case.

Figure 2 shows the effect of each factor separately, i.e., the nonisothermicity and injection, for the argon and the helium, provided that $b_G = 2.0$.

We see that the enthalpy factor ψ_1 in the case of argon injection is reduced in comparison to the erosion case without additional injection, and this in turn increases the erosion by a factor of 1.2.

However, in the case of helium injection the enthalpy factor diminishes the mass erosion by a factor of approximately 2.2 in comparison with the mass erosion that takes place without additional injection. However, the relative contribution of the injection in each of these cases (injection of argon and of helium), calculated from the expression

$$\Psi_{\mathrm{inj}} = \left(1 - \frac{b_1 \Psi_h}{b_{\mathrm{cr}}}\right)^2, \tag{22}$$

is virtually identical and equal to 0.45.

The magnitude of the injected gas at the plate is calculated from the equation

$$j_G = 0.029\; b_G \gamma_0 u_0\, \mathrm{Re}_x^{-0.2}\, \mathrm{Sc}^{-0.6}\, \Psi_h^{0.8} (1 + b_1)^{-0.2} \left(\frac{\mu_w}{\mu_0}\right)^{0.2}. \tag{23}$$

Thus the magnitude of the worn wall material depends on the overall wall-permeability parameter b_1. With an increase in the flow rate for the injected inert gas the mass flow rate of the entrained surface material increases.

Injection of gases with a molecular weight that is smaller in comparison with the basic flow makes it possible to reduce wall erosion both as a consequence of nonisothermicity and as a result of the injection effect.

NOTATION

St_g	is the Stanton diffusion number;
Re_g^{**}	is the Reynolds number which is determined from the thickness of the mass that is lost;
Re_x	is the Reynolds number along the length of the boundary layer;
δ_g^{**}	is the mass-loss thickness;
j_{wall}	is the transverse mass flow resulting from the chemical erosion of the wall material;
j_g	is the transverse mass flow produced by gas injection;
j_w	is the total transverse wall flow;
K_i^0	is the reduced weight concentration of the i-th component of the mixture;
ρ	is the density;
u	is the velocity component along the length of the boundary layer;
v	is the velocity component normal to the motion;
x	is the length of the boundary layer;
ΔK_i^0	is the difference between the reduced concentrations of the i-th component in the main flow and at the wall;
$\bar{x}$	is the dimensionless length of the boundary layer;
D_{12}	is the diffusion factor for the effective binary mixture;
δ	is the boundary-layer thickness;
b_1	is the overall wall-permeability parameter;
b_{wall}	is the wall permeability parameter which is reduced by the lateral flow resulting from the chemical erosion of the wall material;
b_{gas}	is the permeability parameter for the wall subjected to injection;
μ	is the coefficient of dynamic viscosity;
ν	is the coefficient of kinematic viscosity;
Sc	is the Schmidt number;
b_{cr}	is the boundary-layer separation parameter, referred to the Stanton number under standard conditions;
$\gamma_0 u_0$	is the mass flow rate of the main flow;
ψ_1, ψ_2, ψ_3	are, respectively, the ratios of the enthalpies, the molecular weights, and the heat capacities;
h	is the total enthalpy of the gas mixture;
M	is the molecular weight of the mixture;
c_p	is the heat capacity of the mixture;
n_i	is the molar fraction of the i-th component of the gas mixture;
T	is the absolute temperature.

Subscripts

w	denotes the gas parameters at the wall;
0	denotes the core of the flow and the oxidizer;
wall	denotes the wall material;
G	denotes the gas.

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